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Target-space Duality in Heterotic and Type I Effective Lagrangians

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Abstract

We study the implications of target-space duality symmetries for low-energy effective actions of various four-dimensional string theories. In the heterotic case such symmetries can be incorporated in simple orbifold examples. At present a similar statement cannot be made about the simplest type IIB orientifolds due to an obstruction at the level of gravitational anomalies. This fact confirms previous doubts concerning a conjectured heterotic-type IIB orientifold duality and shows that target-space symmetries can be a powerful tool in studying relations between various string theories at the level of the effective low-energy action. Contraints on effective Lagrangians from these symmetries are discussed in detail. In particular, we consider ways of extending T-duality to include additional corrections to the Kähler potential in heterotic string models with $N = 2$ subsectors.

1 Introduction

The idea of the string-driven unification of fundamental interactions has been given a new perspective with the discovery of the web of links, usually referred to as dualities, possibly connecting different string theories. Evidence in favour of the existence of these links has been accumulated through the comparison of various compactifications of different ten-dimensional models and of eleven-dimensional supergravity. However, for the application of these new developments to phenomenologically relevant models in four dimensions, progress has been made in two specific areas. First, the low-energy limit of the strongly coupled heterotic $E_8 \times E_8$ superstring and properties of its four-dimensional effective Lagrangian have been worked out in some detail [1]–[21]. Second, the novel class of four dimensional chiral type I models has been constructed through the compactification of the type IIB string theory on six-dimensional orientifolds [22]–[34].

The type I–heterotic duality in ten dimensions of Polchinski and Witten [35], which exchanges the strongly coupled sector of one theory with the weakly coupled sector of the other, suggests then the existence of links between heterotic and type I models in lower dimensions. In addition, it has been realized that the volume of the compact six-dimensional space appears as a new parameter relevant for duality symmetries. This allows the possibility that heterotic–type I duality can link weakly coupled models on one side to weakly coupled models on the other side in four dimensions. This has even lead to the concept of a generalized duality between heterotic orbifolds and type IIB orientifolds [24], relating models with different numbers of antisymmetric tensor fields. And indeed, candidate dual models in the compactifications of the heterotic $SO(32)$ string have been found. However, since the underlying string theories look quite different, a detailed comparison between type IIB orientifolds and heterotic models is needed either to establish the conjectured duality firmly, or to diagnose points where it breaks down.

One obvious test of the 4d heterotic-type IIB orientifold duality is related to the presence of anomalous local $U(1)$ symmetries. In orientifold models several independent anomalous $U(1)$ factors may be present, whereas in the explicitly known models on the heterotic side there is always only a single anomalous $U(1)$. Attempts to understand the physics of the presumably dual models with anomalous abelian factors have shed new light on the above duality conjecture, and in fact have lead us to identify certain doubts on the validity of 4d heterotic-type IIB duality [34]. A nontrivial test of duality at the level of the 4d effective Lagrangians is associated with isometries of the 4d moduli spaces of prospective dual partners. The experience from the heterotic superstring models tells us that very often such isometries can be extended to symmetries of the full classical effective Lagrangian. In the heterotic string models the best known example of these symmetries are target-space dualities, which are reflections of the underlying symmetry of string theory. As such, target-space dualities are expected to be good quantum symmetries of the effective Lagrangian, which means that currents associated with them should be free of triangle gauge and gravitational anomalies. The requirement of exact quantum T-duality turns out to be a powerful tool in studies of the four-dimensional string models. This symmetry restricts tree-level couplings in the effective action, and in addition allows us to determine the structure of one-loop corrections to that action. This can be shown to be the case of threshold corrections to the gauge couplings in orbifold models. In the present paper we use target-space duality in this spirit, to generalize the form of one-loop corrections to the Kähler potential in the heterotic Lagrangian.

To decide whether the 4d heterotic–type IIB orientifold duality holds, one has to understand what exactly happens to target-space duality in the type IIB orientifold models. In addition, if target-space duality is there in the type IIB orientifold models, this might help to reconstruct the form of their effective Lagrangians, which is crucial from the point of view of phenomenological applications. The problem of realization of target-space duality as a quantum symmetry in effective Lagrangians which might describe type IIB orientifold models in four dimensions is discussed in detail in section 3 of the present paper. We show that even in the simplest models with only D9-branes, it is impossible to enforce cancellation of both gauge and gravitational target-space duality anomalies by a Green-Schwarz mechanism. Furthermore, even if one disregards the gravitational duality anomalies, the structure of the recently computed threshold corrections [36] is not compatible with the Green-Schwarz cancellation of gauge duality anomalies and seems to indicate that target-space duality does not hold at the one-loop level in orientifold models.

However, there is more to say about the one-loop effective Lagrangian. In section 4 we examine in detail specific heterotic models with respect to T-duality invariance. We explain that further corrections to the effective one-loop Lagrangian or to target-space duality transformations must arise in models with a plane fixed under some of the orbifold twists. This necessity is due to those one-loop corrections to the Kähler potential which come together with the well known holomorphic threshold corrections in models with $N = 2$ subsectors. The corrections that we discuss were not taken into account in the earlier analysis of modular anomalies. In the version of the effective Lagrangian, where only holomorphic thresholds are corrected to become covariant with respect to T-duality, the nonholomorphic corrections argued for in section 4 violate T-duality severely. They do it in a way that cannot be easily repaired without introducing additional kinetic mixing between S and T moduli. We propose such a modification, which generalizes the nonholomorphic corrections to the form which is invariant over the full range of the values of the T modulus. The proper inclusion of the nonholomorphic corrections discussed in section 4 may modify somewhat phenomenological implications of the well-known heterotic models.

2 Effective Lagrangians in heterotic models

2.1 Classical symmetries of moduli space

Dimensional reduction of the ten-dimensional supergravity gives a nontrivial kinetic Lagrangian for the universal dilaton and geometric moduli superfields, which is invariant under the action of various symmetry transformations. Geometry of the moduli space is reflected by the Kähler potential

$$K = -\log(S + \bar{S}) - \sum_{i=1}^{3} \log(T_i + \bar{T}_i) \tag{1}$$

There are two obvious symmetries[1] of the kinetic part of the Lagrangian obtained by using this K (for simplicity we put $T_1 = T_2 = T_3 = T$ in the rest of this section):

$$T \rightarrow \frac{aT - ib}{icT + d} \qquad \text{and} \qquad S \rightarrow \frac{\tilde{a}S - i\tilde{b}}{i\tilde{c}S + \tilde{d}}. \tag{2}$$

The first symmetry, target-space duality, is believed to be, in its discrete form, the target-space version of a symmetry of the underlying string theory. The second is broken down at the level of the perturbative Lagrangian to an axionic shift through the coupling to the gauge fields. This target-space S-duality may be restored by nonperturbative effects, but this issue lies beyond the scope of the present paper.

To make the target-space T-duality (2) the classical symmetry of the supergravity Lagrangian [37, 38, 39, 40], one needs to transform the superpotential, W, as well

$$K \rightarrow K + 3\log(icT + d) + \, h.c. \; , \; W \rightarrow (icT + d)^{-3} W \tag{3}$$

If one switches off all the matter and nonuniversal moduli superfields, the suitable form of the superpotential is $W(T) = \frac{const}{\eta^6(T)}$ where η^2 is the squared Dedekind's modular form of weight one. Further, it is possible to extend this clasical symmetry of the tree-level Lagrangian to include also matter and nonuniversal moduli fields. These fields transform as tensors, i.e. linearly, and their entries in the Kähler potential are modular invariant on their own (see the Appendix). For instance, the Kähler potential for untwisted matter is $K_A = A\bar{A}/(T + \bar{T})$, for twisted moduli C it turns out to be $K_C = C\bar{C}/(T + \bar{T})^3$, and for twisted matter A_C $K_{A_C} = A_C\bar{A}_C/(T + \bar{T})^2$ (like in the Z_3 orbifold example).

2.2 Cancellation of one-loop anomalies

Target-space duality transformations involve sigma-model transformations, and we have to check whether they are anomalous at the one-loop order [41], [42], [43]. By the term sigma model we understand the supersymmetric sigma model defined through the kinetic terms of the form $g_{i\bar{j}}(T,\bar{T})\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{j}}$ where $g_{i\bar{j}} = \partial K/\partial\phi^i\partial\bar{\phi}^{\bar{j}}$ with the Kähler potential[2] K. The form of the sigma model metric $g_{i\bar{j}}$ for indices corresponding to various fields was given at the end of the previous subsection. We can see, that upon T-duality transformations the form of $g_{i\bar{j}}$ changes, and to compensate for that change one needs to 'rotate' the ϕ's. These rotations, due to supersymmetry, act also on all fermions present in the model, and transform them in the way of chiral rotations, with charges related to the modular weights of the fields (see the Appendix). This chiral transformation results in an anomaly in the divergence of the current associated with T-duality. In addition, under T-duality the Kähler potential is not invariant, but suffers a shift of the form $F(T) + \bar{F}(\bar{T})$ where F is holomorphic. This shift is absorbed in the redefinition of the superpotential W, which in turn cancels against the transformation (3) of W. Such Kähler transformation also results in the rotation of chiral fermions, this time with the same charge for all chiral multiplets and just the opposite one for gauginos. This

[1]The symmetries which contain as a subset the invariances of the kinetic Lagrangian of the moduli we shall refere to as sigma model symmetries.

[2]We suppress the dependence on the modulus S as at tree level S is inert under the T-duality transformations.

rotation is also anomalous and the anomaly must be taken into account in addition to the sigma-model anomaly. The anomalous diagrams can be visualized as triangle diagrams with two gauge bosons or two gravitons and one composite connection which plays the role of the gauge field of T-duality. In fact, the part of the composite connection whose T-duality variation leads to nonvanishing variation of the diagrams is $V_\mu \sim \frac{\partial_\mu(T-\bar{T})}{T+\bar{T}}$, since one typically assumes that all other non-inert fields fluctuate around vanishing vacuum values, hence the anomalous graphs are simply these with $\partial_\mu Im(T)$ at one vertex and gauge bosons or gravitons at remaining vertices. The sigma model anomalies can be computed in field theory limit of string theories, and should be cancelled for the sake of the quantum exactness of the target-space dualities. The form of the one-loop terms needed in the effective Lagrangian to cancel anomalies can be worked out in field theory limit, but these terms can also be directly computed in string theory in various orbifold models, and there they have simply the status of one-loop corrections to the effective action, with their coefficients given from string theory. In the rest of this subsection we recapitulate briefly the status of these corrections in relevant classes of orbifold models.

Let us start with the the simplest cases when all the orbifold planes are rotated by the orbifold twists (e.g. the Z_3 and Z_7 orbifolds). Then there are neither holomorphic threshold corrections, nor the associated corrections to the Kähler potential that will be described in section 4, and all T-duality anomalies, gauge and gravitational, are cancelled through the universal four dimensional Green-Schwarz mechanism. More precisely, the 1-loop anomalies get cancelled by the shift of the dilaton superfield S: $S \to S - \frac{3\delta_{GS}}{8\pi^2}\log(icT+d)$. Then the dilaton Kähler potential is modified to $K = -\log(S+\bar{S} - \frac{3\delta_{GS}}{8\pi^2}\log(T+\bar{T}))$ which is T-duality invariant. The details of that cancellation are given in the Appendix. This four-dimensional Green-Schwarz mechanism [44], [45], [46] is sufficient to cancel all anomalies in models as e.g. the Z_3 and Z_7 orbifolds.

In orbifold models with invariant planes, which contain N=2 subsectors, anomalies are no longer universal and one needs additional counterterms to cancel them completely [41]. These counterterms depend on moduli in a holomorphic way, and they are interpreted as holomorphic one-loop corrections to the tree-level gauge kinetic functions. In the most general case, using eq. (A.6), (A.9) and (A.11) from the Appendix, one can rewrite the one-loop effective lagrangian for the gauge fields as[3]:

$$\begin{aligned}\mathcal{L}_{GK} &= \frac{1}{4}\sum_a \int \mathrm{d}^2\theta\, W^a W^a\, P_C \Bigg\{ \Bigg[S+\bar{S} + \sigma + \bar{\sigma} - \frac{1}{8\pi^2}\sum_i \delta^i_{GS}\ln(T_i+\bar{T}_i) \Bigg] \\ &\quad - \frac{1}{8\pi^2}\sum_i (b'^i_a - \delta^i_{GS})\ln\Big[\,|\eta(T_i)|^4 (T_i+\bar{T}_i)\Big] \Bigg\} + \text{h.c.} \end{aligned} \tag{4}$$

where $\sigma(T)$ is the holomorphic part of the universal one-loop threshold correction invariant under $SL(2,Z)$ T-duality transformations and approaching $-\frac{1}{4\pi}T$ when $T \to \infty$. The two terms in the bracket are separately modular invariant. From this expression one can extract the evolution of gauge couplings, upon adding the field-independent terms due to loops of massless charged states:

$$\frac{1}{g_a^2}\Big|_{\text{1-loop}} = \mathrm{Re}S + \mathrm{Re}\sigma - \frac{1}{16\pi^2}\sum_i \delta^i_{GS}\ln(T_i+\bar{T}_i) - \frac{b_a}{16\pi^2}\ln\frac{\mu^2}{M_H^2}$$

[3]For simplicity we suppress the dependence of the threshold corrections on the complex structure moduli U.

$$- \frac{1}{16\pi^2} \sum_i b_{a,i}^{N=2} \ln \left[|\eta(T_i)|^4 (T_i + \bar{T}_i) \right] \tag{5}$$

where M_H is the (heterotic) string scale. This expression reflects already the knowledge of the explicit string computations of the nonuniversal threshold corrections to gauge couplings, as the coefficients $b_{a,i}^{N=2}$ are one-loop beta functions coming from $N = 2$ subsectors associated with invariant planes. The holomorphic part of the universal threshold correction – $\sigma(T)$ – remains to be determined for arbitrary T via string calculations.

String results are naturally formulated in the linear multiplet formalism (see the Appendix), since the string degrees of freedom are antisymmetric tensor fields themselves and not the pseudoscalars dual to them. Later in this paper we shall try to find relations between these string degrees of freedom and the chiral multiplets suitable for low-energy supergravity description of the type IIB orientifold models. Here we merely point out that the expression of the one-loop string coupling $1/l$ in terms of the effective supergravity moduli fields ReS and ReT_i (the one-loop chiral-linear duality relation (B.4)) is determined by the cancellation of sigma-model anomalies (see the Appendix).

We can now move on to the discussion of the Z_3 and Z_7 type IIB orientifold compactifications and their relation to heterotic compactifications.

3 Target-space duality in $D = 4$, $N = 1$ type IIB Z_N orientifolds

Let us consider orientifolds [22, 23] of type IIB string theory compactified on a T^6/Z_N orbifold [24]–[31]. Consistency of these models (absence of RR tadpoles that would spoil the UV finiteness of the theory) requires the introduction of D-branes on which open strings can end. If the orientifold projector is chosen to be the world-sheet parity Ω, one can have either only D9-branes or both D9-branes and D5-branes. In addition, not all twists leading to $N = 1$ supersymmetry are allowed by tadpole conditions (for a classification see ref. [30]); the consistent Z_N orientifolds that contain only 9-branes are the odd N (Z_3 and Z_7) orientifolds.

It has been noticed [47] that the classical Lagrangian of orientifolds with only D9-branes is invariant under $SL(2, R)_{T_i}$ transformations. Indeed, the effective Lagrangian describing the dynamics of the open string and untwisted closed string sectors of these models, which can be obtained by reduction and truncation from the $D = 10$ type I supergravity action, has the same structure as the Lagrangian of the untwisted sector of heterotic orbifolds:

$$f_9 = S , \qquad W \sim C_1^9 \, C_2^9 \, C_3^9 , \tag{6}$$

$$K = - \ln(S + \bar{S}) - \sum_i \ln \left(T_i + \bar{T}_i + |C_i^9|^2 \right) , \tag{7}$$

where f_9 denotes the gauge kinetic function for gauge fields of the D9-brane sector, and C_i^9 is a generic (99) matter field associated with the i^{th} complex plane. The addition of the twisted closed string states (which are gauge singlets) does not spoil this invariance. In models containing 5_i-branes (5-branes wrapping on the i^{th} compact plane), $SL(2, R)_{T_i}$ is explicitly broken by the gauge kinetic function of the $(5_i 5_i)$ gauge bosons, $f_{5_i} = T_i$, but modular invariance with respect to the j^{th} compact plane still holds as soon as no 5_j-branes are present [47].

One may then ask whether target-space modular invariance is a good quantum symmetry of $D = 4$, $N = 1$ type IIB orientifolds. Although it is not clear what would be the origin of this symmetry in the underlying string theory[4], it is expected to hold at the one-loop level if heterotic - type IIB duality is valid, because then sigma-model anomalies are compensated for in the dual heterotic orbifold models (strictly speaking, this argument does not apply to orientifolds containing 5-branes, since these models do not seem to have any perturbative heterotic dual). Thus investigating the possibility of cancelling sigma-model anomalies in orientifolds amounts to testing duality. Also, the question of whether target-space modular invariance is a good quantum symmetry in orientifolds is interesting on its own, even if this string duality does not hold, because it could give us information on the effective Lagrangian of those models, which are not so well known. The purpose of this section is to investigate this question at the level of the effective field theory, using the information given by recent string computations.

The authors of ref. [47] have proposed a mechanism for the cancellation of sigma-model anomalies that is reminiscent of the way Abelian gauge anomalies are compensated for in orientifolds [48]. The gauge group of orientifold models often contains several anomalous Abelian factors $U(1)_i$. Their anomalies are cancelled by a generalized [49] Green-Schwarz mechanism involving RR twisted antisymmetric tensors $B^k_{\mu\nu}$ with appropriate couplings to the gauge fields. In a more familiar chiral superfield language, those antisymmetric tensors are described by their pseudoscalar duals a_k, which lie in the same chiral multiplets M_k as the scalars corresponding to the blowing-up modes of the orientifold, $a_k = \mathrm{Im} M_k \,|_{\theta=\bar\theta=0}$. The basic ingredients of the generalized Green-Schwarz mechanism are a coupling of the M_k to the gauge fields,

$$f_a \;=\; f_p \;+\; \sum_{k=1}^{[\frac{N-1}{2}]} s_{ak}\, M_k \ , \tag{8}$$

with $f_p = S$ for gauge group coming from 9-branes, and a shift of the twisted axions a_k under a $U(1)_i$ gauge transformation:

$$M_k \;\to\; M_k \,+\, i\, \delta^k_i \Lambda_i \ . \tag{9}$$

In eq. (8), the sum goes over independent twisted sectors, and for a twist θ^k with no fixed plane, the M_k fields are defined by $M_k = \frac{1}{\sqrt{N_k}} \sum_{f=1}^{N_k} M^f_k$, where $\{M^f_k\}_{f=1 \ldots N_k}$ are the states from the k^{th} twisted sector, each of them living at one of the N_k points that are fixed under θ^k (for a twist leaving some plane unrotated, not all twisted states fit into linear multiplets, see ref. [51]). It is interesting to note that the shift (9) is a one-loop effect in the low-energy effective field theory, although its string origin is a tree-level Green-Schwarz coupling at the level of the orientifold. From eq. (8) and (9) one obtains that mixed $U(1) G_a G_a$ anomalies are cancelled if the following conditions are fulfilled:

$$C^i_a \;=\; 8\,\pi^2 \sum_k c^k_a\, \delta^k_i \ . \tag{10}$$

[4]In heterotic compactifications, discrete modular transformations of the T_i moduli correspond to a fundamental quantum string symmetry, T-duality. On the contrary, there is no such connection between the target-space modular invariance observed at the level of the effective Lagrangian of orientifolds and T-duality, since the latter exchanges Dirichlet and Neumann boundary conditions, and therefore does not leave a given D-brane configuration invariant.

This Green-Schwarz cancellation of $U(1)$-gauge anomalies has been confirmed by a string computation of the couplings s_{ak} and δ_i^k [36]. Since the absence of $U(1)$-gravitational anomalies is also required for the model to be consistent, it is natural to assume [47] that they are compensated for by the same mechanism (this proposal has been confirmed in the recent paper [52]), i.e. that the twisted axions a_k couple to the $R\widetilde{R}$ term:

$$-\frac{1}{4}\,(a + t_k\, a_k)\; R\widetilde{R}\ . \tag{11}$$

$U(1)$-gravitational anomalies must then satisfy the conditions:

$$\frac{\mathrm{Tr} X_i}{12}\ =\ 8\,\pi^2 \sum_k\ t_k\, \delta_i^k\ . \tag{12}$$

Contrary to the s_{ak}, the couplings t_k have not been computed yet; however, they can be extracted from eq. (12), since the δ_i^k are known from ref. [36].

The authors of ref. [47] have proposed that sigma-model anomalies associated with complex planes that are rotated by all twists are cancelled by a similar Green-Schwarz mechanism. The most general possibility is that both S and the M_k shift under $SL(2,R)_{T_i}$ transformations: $S \to S - \frac{1}{8\pi^2}\,\delta_{GS}^{i,S} \ln(ic_iT_i + d_i)$, $M_k \to M_k - \frac{1}{8\pi^2}\,\delta_{GS}^{i,k} \ln(ic_iT_i + d_i)$. The sigma-gauge anomalies must then satisfy:

$$b_a'^i\ =\ \delta_{GS}^{i,S}\ +\ \sum_k\ s_{ak}\,\delta_{GS}^{i,k}\ . \tag{13}$$

Since there are generically as many Green-Schwarz parameters $\delta_{GS}^{i,S}$ and $\delta_{GS}^{i,k}$ as anomalies, it is always possible to cancel the sigma-gauge anomalies by means of a Green-Schwarz mechanism. Given the couplings s_{ak}, which are known from ref. [36], and the anomaly coefficients $b_a'^i$, which are computed from the massless spectrum, eq. (13) determines uniquely the Green-Schwarz parameters that are needed in order to ensure anomaly cancellation. One finds [47] $\delta_{GS}^{i,S} = 0$ for all Z_N orientifolds of ref. [30], i.e. the dilaton does not play any role in the cancellation of sigma-gauge anomalies. Although this last feature, which is reminiscent of the mechanism of Abelian gauge anomaly cancellation, may appear to be suggestive, it should be stressed that it is not possible to check the conjecture of a Green-Schwarz mechanism on the basis of an analysis of the mixed gauge anomalies, since there are as many potential counterterms as anomalies. This situation is to be contrasted with the case of heterotic orbifolds with no fixed planes, in which three parameters δ_{GS}^i, $i = 1,2,3$ must cancel all anomalies, thus implying several consistency relations that can be checked in explicit models (namely the anomalies $b_a'^i$ must be gauge-group independent).

3.1 Sigma-gravitational anomalies

However, one can test this conjecture by considering sigma-gravitational anomalies. Indeed, any shift of the universal and/or twisted axions under target-space modular transformations induces, through the couplings (11), a variation of the Lagrangian $\delta\mathcal{L} = \frac{\theta_i}{768\pi^2}\,\bar{b}_{grav}'^i R\widetilde{R}$ that cancels part of the triangle anomaly $b_{grav}'^i$. Assuming that the sigma-gauge anomalies are compensated for by a Green-Schwarz mechanism, the shifts of the axions a and a_k under $SL(2,R)_{T_i}$ are determined in an unambiguous way by eq. (13), therefore the corresponding coefficient $\bar{b}'^i_{grav}$ read, given the fact that $\delta^{i,S}_{GS} = 0$ in all cases:

$$\bar{b}'^i_{grav} \;=\; 24 \sum_k t_k \, \delta^{i,k}_{GS} \tag{14}$$

If a Green-Schwarz mechanism ensures the validity of target-space duality at the one-loop level, then one should have $b'^i_{grav} = \bar{b}'^i_{grav}$. It is straightforward to check this relation in explicit models. For definiteness we restrict ourselves to the odd order Z_N orientifolds, which contain only 9-branes and do not have any fixed plane. The values of the triangle anomaly b'^i_{grav} and of the Green-Schwarz contribution $\bar{b}'^i_{grav}$ are displayed in Table 1, where b'^i_M denotes the contribution of the M_k fields to b'^i_{closed}; since they transform non-linearly under $SL(2,R)_{T_i}$, they have zero modular weights and b'^i_M is just the number of M_k fields ($b'^i_M = 27$ and 21 for Z_3 and Z_7 respectively). Following ref. [47], we also give, for later discussion, the separate contributions of the open and closed string states to the triangle anomaly:

$$b'^i_{open} \;=\; -\ \dim G \;+\; \sum_\alpha (1 + 2n^i_\alpha) \ , \tag{15}$$

$$b'^i_{closed} \;=\; 21 \;+\; 1 \;+\; b'^i_{mod} \ , \tag{16}$$

where b'^i_{mod} denotes the contribution of the other modulinos than the dilatino, and by definition $b'^i_{grav} = b'^i_{open} + b'^i_{closed}$.

model	b'^i_{open}	b'^i_{closed}	b'^i_{grav}	$\bar{b}'^i_{grav} \equiv 24 \sum_k t_k \delta^{i,k}_{GS}$
Z_3	-10	$19 + b'^i_M$	$9 + b'^i_M$	-18
Z_7	-6	$21 + b'^i_M$	$15 + b'^i_M$	-6

Table 1

From Table 1 it is obvious that $\bar{b}'^i_{grav} \neq b'^i_{grav}$, i.e. the Green-Schwarz mechanism alone cannot compensate for both sigma-gauge and sigma-gravitational anomalies. However, there may exist some other effect that would cancel the remaining anomaly $b'^i_{grav} - \bar{b}'^i_{grav}$. This is actually suggested by the string diagramatics relevant for sigma-gravitational anomalies in open string models, as explained in ref. [47]. Indeed, only diagrams with an open string loop (the annulus and Moebius amplitudes) can contribute to sigma-gauge anomalies (as well as $U(1)$ anomalies), but in the case of sigma-gravitational anomalies, there are additional contributions coming from diagrams with a closed string loop (the torus and Klein bottle amplitudes). Such diagrams are not present for sigma-gauge anomalies, because they would be of higher order in the string coupling. The statement that sigma-gravitational anomalies are cancelled is equivalent to the statement that all four diagram topologies should sum up to zero, and that the field theory limit of these string diagrams should contain both the triangle anomaly (which one can split into two separate contributions, b'^i_{open} and b'^i_{closed}) and the corresponding counterterms. The open string diagrams are expected to provide the field theory anomalous tree graphs that are characteristic of a Green-Schwarz mechanism, since these diagrams, when considered in the closed string (tree) channel, are suggestive of a factorized form with closed string states propagating in the cylinder (from the cancellation of sigma-gauge anomalies, we know that those states must be twisted RR antisymmetric tensors). The interpretation of the closed string diagrams is less obvious. The authors of ref. [47] assume that factorization is possible and that these diagrams generate a one-loop mixing between the dilaton and the T moduli that make up the sigma-model connection. However, this proposal appears to be incompatible with the above field theory analysis, since it would imply a one-loop[5] shift of the universal axion under $SL(2,R)_{T_i}$ transformations, which has already been excluded on the basis of sigma-gauge anomaly cancellation (the same conclusion would hold for an additional mixing between the twisted M_k fields and the T moduli). This suggests that the possible counterterms originating from closed string loops are not Green-Schwarz terms. The only alternative possibility we can think of would be a T_i-dependent correction to the CP-odd R^2 terms with the appropriate behaviour under $SL(2,R)_{T_i}$, i.e.

$$\mathcal{L}_{CT} \;=\; \frac{1}{32\pi^2}\,\mathrm{Im}\,\Delta(T)\,R\widetilde{R}\;, \qquad \Delta(T) \;\stackrel{SL(2,R)_{T_i}}{\longrightarrow}\; \Delta(T) \;+\; \frac{b'^i_{grav}-\bar{b}'^i_{grav}}{24}\;\ln(ic_iT_i+d_i)\;. \tag{17}$$

However, such corrections are not expected in models which do not possess any $N=2$ sectors. Moreover, they cannot appear at the perturbative level because of the Peccei-Quinn symmetries associated to the T_i axions. Still one cannot exclude the possibility that they are generated nonperturbatively.

Before concluding this subsection, let us comment more precisely on the concrete proposal of ref. [47]. There it was assumed that the contribution of the open string sector to sigma-gravitational anomalies, b'^i_{open}, was exactly compensated for by the $SL(2,R)_{T_i}$ shift of the M_k moduli, whereas b'^i_{closed} was taken care of by closed string loop diagrams (interpreted as a mixing between the dilaton and the T moduli, which as explained above seems to be strongly disfavoured by the field theory analysis). If this picture were correct, one would find $b'^i_{open}=\bar{b}'^i_{grav}$ in all explicit orientifold models. However, Table 1 shows that it is not verified in the Z_3 case, although it accidentally holds in the Z_7 case.

We are therefore led to the following conclusion: either modular invariance is not a good quantum symmetry in orientifold models, which casts a new doubt on the validity of heterotic - type IIB duality; or it is a good quantum symmetry and the anomalies cannot be cancelled by a pure Green-Schwarz mechanism, even in models that do not possess any fixed planes. One would then require T_i-dependent, holomorphic corrections to the R^2 terms, which however cannot arise perturbatively.

[5] Although it has been argued [47] on the basis of string diagramatics that a one-loop mixing between the dilaton and the T moduli should not affect sigma-gauge anomalies, because it would give a higher order contribution, this is not the case in the effective field theory, where such a mixing would yield terms proportional to $F^a\widetilde{F}^a$ in the variation of the Lagrangian, together with the $R\widetilde{R}$ piece.

3.2 One-loop corrections to the gauge couplings

As stressed above, the consideration of field theory anomalies is not enough to decide whether target-space duality is a good quantum symmetry of type IIB orientifolds, mainly because of the large number of possible counterterms. Although the cancellation of sigma-gravitational anomalies already appears to be problematic, further tests of this symmetry are needed. The one-loop corrections to gauge couplings that have been recently computed in type IIB orientifolds [36] give us the opportunity to perform such a test. Indeed, the string result should agree with the one-loop corrections computed in the effective supergravity theory. While the former depends on the linear multiplets of the theory, the latter is expressed in terms of chiral multiplets only; a duality transformation relates the two formulations. The relations between the linear multiplets and their dual chiral multiplets are defined order by order in perturbation theory. As is well known from the heterotic case[6], the presence of Green-Schwarz counterterms modifies these relations at the one-loop order (see eq. (B.4) in Appendix B). In the following, we shall then try to relate the string and supergravity expressions for the one-loop gauge couplings through an explicit linear-chiral duality transformation. The duality relations obtained in this way should tell us whether the couplings needed for the cancellation of sigma-model anomalies are indeed generated in orientifold models.

Let us first establish the tree-level duality relations. The linear multiplets we have to deal with are the universal linear multiplet $L \sim (l, B_{\mu\nu}, \chi)$ and a model-dependent number of twisted linear multiplets $L_k \sim (m_k, B_{k\mu\nu}, \chi_k)$, which describe the dilaton and the blowing-up modes, respectively, as well as their antisymmetric tensor partner. The tree-level Lagrangian for L and L_k has to reproduce the tree-level gauge couplings (here we consider only gauge groups coming from the 9-brane sector) [36]

$$\frac{1}{g_a^2} \;=\; \frac{1}{l} \;+\; \sum_k s_{ak}\, m_k \;. \tag{18}$$

Assuming no kinetic mixing between the different linear multiplets (this is indeed the case in the basis in which formulae (18) is written), one finds:

$$\mathcal{L} \;=\; \int \mathrm{d}^4\theta \;\Phi\,(\widehat{L}, \widehat{L}_k) \;, \qquad \Phi\,(\widehat{L}, \widehat{L}_k) \;=\; \ln \widehat{L} \;-\; \sum_k \widehat{L}_k^2 \;, \tag{19}$$

where $\widehat{L} = L - 2\,\Omega$ and $\widehat{L}_k = L_k + \sum_a s_{ak}\Omega_a$ are the modified, gauge invariant multiplets. Following the standard procedure (see Appendix B), one then obtains the tree-level duality relations:

$$\frac{1}{\widehat{L}} \;=\; \frac{S+\bar{S}}{2} \;, \qquad \widehat{L}_k \;=\; \frac{M_k + \bar{M}_k}{2} \;, \tag{20}$$

as well as the Kähler potential and gauge kinetic function that define the Lagrangian describing the dual chiral superfields S and M_k:

$$f_a \;=\; S \;+\; \sum_k s_{ak}\, M_k \;, \qquad K(S, \bar{S}, M_k, \bar{M}_k) \;=\; -\,\ln(S+\bar{S}) \;+\; \frac{1}{4}\,(M_k + \bar{M}_k)^2 \;. \tag{21}$$

[6]Green-Schwarz cancellation of sigma-model anomalies in the presence of several linear multiplets L and L_k has been described in detail in ref. [51].

One recovers, as expected, the gauge kinetic function (8). Note that, since we started from eq. (18), which is the first order in a perturbative expansion around the orientifold point $m_k = 0$, eq. (19), (20) and the Kähler potential in eq. (21) are valid at leading order in M_k only.

At the one-loop level, the Lagrangian (19) receives corrections which modify the duality relations (20) and the Lagrangian in the chiral basis (21). The loop-corrected Lagrangian in the linear basis (respectively the chiral basis) should reproduce the one-loop gauge couplings computed in ref. [36] (respectively the one-loop gauge couplings computed in the effective supergravity theory [53]). For the sake of simplicity, we restrict our discussion to the case of odd N Z_N orientifolds. These models do not possess any $N=2$ sectors, so that the only way to compensate for sigma-model anomalies is the Green-Schwarz mechanism, as discussed in the previous subsection. The one-loop gauge couplings obtained from the string computation of ref. [36] read:

$$\frac{1}{g_a^2(\mu^2)}\Bigg|_{\text{1-loop}} = \frac{1}{l} + \sum_k s_{ak}\, m_k - \frac{b_a}{16\pi^2} \ln \frac{\mu^2}{M_I^2} \ . \tag{22}$$

The generic expression for one-loop gauge couplings in effective supergravity theories read [53]:

$$\begin{aligned} \frac{1}{g_a^2(\mu^2)}\Big|_{\text{1-loop}} &= \text{Re} f_a \,\Big|_{\text{1-loop}} - \frac{b_a}{16\pi^2} \ln \frac{\mu^2}{M_{pl}^2} \\ &+ \frac{1}{16\pi^2}\left[c_a K + 2\, C_2(G_a) \ln g_a^{-2} - 2 \sum_\alpha T(R_\alpha) \ln \det Z_{R_\alpha} \right] , \end{aligned} \tag{23}$$

where Z_{R_α} is the effective wave function normalization matrix for the representation R_α, and $c_a = \sum_\alpha T(R_\alpha) - C_2(G_a)$. In the right hand side of eq. (23), the functions K, g_a^{-2} and Z_{R_α} are truncated at tree-level, while $f_a \,|_{\text{1-loop}}$ includes the one-loop string corrections. Since formula (23) is valid in any supergravity effective theory, independently of its string origin, we can apply it to the orienfold models we are considering. Then f_a is given by its tree-level expression (21) (the string threshold corrections vanish in the absence of $N=2$ sectors), $K = K(S,\bar{S}; M_k, \bar{M}_k) - \sum_i \ln(T_i + \bar{T}_i)$ where $K(S,\bar{S}; M_k, \bar{M}_k)$ is given by (21), and $Z_{R_\alpha} = \Pi_i (T_i + \bar{T}_i)^{n^i_{R_\alpha}}$. Eq. (23) then becomes:

$$\frac{1}{g_a^2(\mu^2)} = \text{Re}S + \sum_k s_{ak}\, \text{Re}M_k - \sum_i \frac{b'^i_a}{16\pi^2} \ln(T_i + \bar{T}_i) - \frac{b_a}{16\pi^2} \ln \frac{\text{Re}S\, \mu^2}{M_{pl}^2} \ , \tag{24}$$

up to terms that are suppressed both by a one-loop factor and a dependence on $\text{Re}M_k$, which we can neglect. Note that the T-dependent non-harmonic corrections (third piece in the right hand side of eq. (24)) are related, through supersymmetry, to the anomalous triangle diagrams associated with the sigma-model connection.

Assuming that it is possible to perform a linear-chiral duality transformation at the one-loop level in a consistent way, one can obtain the one-loop linear-chiral duality relations for $\widehat{L}$ and $\widehat{L}_k$, as was done in ref. [36], by comparing the string expression (22) with the effective supergravity expression (24). This gives:

$$\frac{1}{l} = \text{Re}S \ , \tag{25}$$

$$m_k = \text{Re}M_k - \frac{1}{16\pi^2} \sum_i \delta^{i,k}_{GS} \ln(T_i + \bar{T}_i) - \frac{b_k}{16\pi^2} \ln \left(\frac{\text{Re}S}{\Pi_i\, \text{Re}T_i} \right)^{1/2} , \tag{26}$$

where the coefficients $\delta_{GS}^{i,k}$ are defined by eq. (13) (with $\delta_{GS}^{i,S} = 0$ in all models), and the coefficients b_k are defined by $b_a^{N=1} = \sum_k s_{ak}\, b_k$. The second piece in the right hand side of the duality relation (26) then comes precisely with the coefficients needed for the Green-Schwarz cancellation of the sigma-gauge anomalies; however, the third piece transforms under $SL(2,R)_{T_i}$ in the same way as Green-Schwarz counterterms and seems to spoil anomaly cancellation. The presence of this troublesome term can be traced back to the difficulty of relating the string expression (22) to the supergravity expression (24) through a linear-chiral duality transformation, as we discuss below.

The expression for the one-loop gauge couplings in the linear basis, eq. (22), does not contain any term proportional to $\ln(T_i+\bar{T}_i)$. This means that the one-loop Lagrangian for the linear multiplets must contains a term[7]

$$\Delta\mathcal{L}_{GS} \;=\; -\frac{1}{8\pi^2}\sum_{i,k}\delta_{GS}^{i,k}\,\widehat{L}_k \ln(T_i+\bar{T}_i) \tag{27}$$

whose contribution to the gauge couplings exactly cancels the non-harmonic corrections coming from the supersymmetric partners of the anomaly diagrams, $-\sum_i \frac{b_a'^i}{16\pi^2}\ln(T_i+\bar{T}_i)$. Note that this Green-Schwarz mechanism is not enough to ensure $SL(2,R)_{T_i}$ invariance of the gauge couplings, since the upper scale of logarithmic running, the string scale M_I, is not invariant on its own. Indeed, when expressed in units of the Planck mass, M_I depends on the T_i moduli: $M_I^2 = \frac{\lambda_I M_{Pl}^2}{2\,\mathrm{Re}S} = (\frac{\mathrm{Re}S}{\Pi_i \mathrm{Re}T_i})^{1/2}\frac{M_{Pl}^2}{2\,\mathrm{Re}S}$ (we have used the identities $\mathrm{Re}S = V_I M_I^6/\lambda_I$ and $\mathrm{Re}T_i = V_I^i M_I^2/\lambda_I$ [30], where λ_I is the ten-dimensional string coupling and V_I^i is the volume of the i^{th} compact torus in the string metric). Now the addition of the Green-Schwarz counterterms (27) to the Lagrangian (19) leads to the modified duality relations

$$\frac{1}{\widehat{L}} \;=\; \frac{S+\bar{S}}{2}\,, \tag{28}$$

$$\widehat{L}_k \;=\; \frac{1}{2}\left[M_k+\bar{M}_k \;-\; \frac{1}{8\pi^2}\sum_i \delta_{GS}^{i}\ln(T_i+\bar{T}_i)\right]\,, \tag{29}$$

which do not agree with eq. (26) and therefore yield an expression for the gauge couplings in the chiral basis that does not fit the supergravity expression (24).

One can try to solve this problem by adding to the Lagrangian of the linear multiplets, beyond the terms (27), the piece that reproduces the fitted duality relation (26), i.e.

$$\Delta\mathcal{L}_2 \;=\; \frac{1}{16\pi^2}\sum_k b_k\widehat{L}_k \ln\left[\widehat{L}\,\prod_i(T_i+\bar{T}_i)\right] \tag{30}$$

($\Delta\mathcal{L}_2$ also modifies the duality relation between $\widehat{L}$ and S, but the corrections can be neglected, because it is proportional both to a one-loop factor and to m_k). This reproduces the supergravity expression (24), but looks somewhat ad hoc; in particular, this amounts to shift the upper scale of running to the invariant scale $M_{Pl}^2/\mathrm{Re}S$ and to absorb the residual, non-invariant logarithmic term into the effective one-loop Lagrangian. Note that the non-invariant part of $\Delta\mathcal{L}_2$

[7] As stressed before in the context of Abelian gauge anomalies, the couplings $\delta_{GS}^{i,k}$ arise at the one-loop level in the effective field theory, although they are presumably present at tree level in the orientifold sense.

has the same form as the Green-Schwarz counterterms $\Delta\mathcal{L}_{GS}$; as a result, this new term modifies the shift of the M_k fields under $SL(2,R)_{T_i}$ and only a part of the sigma-model anomalies is cancelled. Indeed, the modified Kähler potential obtained from the duality transformation is a function of the combinations of chiral superfields corresponding to the linear multiplets $\widehat{L}_k$; then requiring invariance of K amounts to modify the shift of the M_k to:

$$M_k \;\to\; M_k \;-\; \frac{1}{8\pi^2}\left(\delta_{GS}^{i,k} - \frac{b_k}{2}\right)\ln(ic_iT_i + d_i)\ . \tag{31}$$

The uncancelled anomaly is $b_a'^i - \sum_k s_{ak}(\delta_{GS}^{i,k} - \frac{b_k}{2}) = b_a/2$. Alternatively, one may absorb the terms (30) into a redefinition of the twisted linear multiplets at the one-loop level, $\widetilde{L}_k = L_k - \frac{b_k}{32\pi^2}\ln[\widehat{L}\,\Pi_i(T_i+\bar{T}_i)]$. The one-loop duality relations are then given by eq. (29), and the Green-Schwarz couplings ensure exact cancellation of sigma-gauge anomalies. However, target-space duality is violated at the level of one-loop corrections to the Kähler potential, due to the fact that the constraints $D^2L_k = D^2(\widetilde{L}_k - \frac{b_k}{32\pi^2}\ln[\widehat{L}\,\Pi_i(T_i+\bar{T}_i)]) = 0$ and $\bar{D}^2L_k = 0$ are no longer invariant.

We conclude that the structure of the one-loop corrections to gauge couplings in type IIB orientifolds does not seem to be compatible with the cancellation of sigma-gauge anomalies by a pure Green-Schwarz mechanism, even in the simplest models with no $N=2$ subsectors, contrary to what the mere analysis of anomalies would suggest. This, together with the difficulty observed at the level of mixed gravitational anomalies, suggests that target-space duality is merely an accidental, tree-level symmetry of the effective supergravity Lagrangian of orientifold models, and strengthen previous doubts [34] on the validity of the heterotic - type IIB duality. Note that it is not completely clear how the agreement between the string and effective supergravity expressions for the one-loop gauge couplings is obtained; in this respect, an explicit string computation of corrections to the twisted moduli Kähler potential would give very useful information.

3.3 Threshold corrections and heterotic - type IIB duality

We have seen that target-space duality can be used as a tool to test heterotic - type IIB duality at the one-loop level. For completeness, we would like to mention another nontrivial check based on the comparison of the threshold corrections in candidate dual orbifold/orientifold models, which also raises doubts on the validity of this duality (the following discussion is taken from ref. [36]). Specifically, we shall consider the Z_3 models, in which a perfect matching of the massless spectra and gauge groups of both low-energy effective field theories is obtained after decoupling of the anomalous $U(1)$. The one-loop gauge couplings of the Z_3 orbifold/orientifold computed in string theory read [36]:

$$\frac{1}{g_a^2(\mu^2)\,|_H} \;=\; \frac{1}{l_H} \;-\; \frac{b_a^H}{16\pi^2}\ln\frac{\mu^2}{M_H^2}\ , \tag{32}$$

$$\frac{1}{g_a^2(\mu^2)\,|_I} \;=\; \frac{1}{l_I} \;+\; s_a\,m \;-\; \frac{b_a^I}{16\pi^2}\ln\frac{\mu^2}{M_I^2}\ , \tag{33}$$

where M_H (respectively M_I) is the heterotic (respectively type I) string scale, and m is the symmetric combination of the 27 twisted scalars. At tree level[8], the $D = 10$ heterotic - type I duality implies $l_H = l_I$, but this relation does no longer hold at the one-loop level. The correct duality relation is found by expressing the linear multiplets in terms of the chiral fields S and T_i, and using the duality dictionary derived by dimensional reduction of the $D = 10$ duality relations [35] $\lambda_H = \lambda_I^{-1}$ and $M_H^2 = \lambda_I^{-1} M_I^2$ (where λ_H, respectively λ_I, is the $D = 10$ heterotic, respectively type I dilaton). One finds $V_H = V_I$, where $V_{H(I)} = \int dx^6 \sqrt{g^{(6)}_{H(I)}}$ is the compact volume in the string metric, as well as[9] $(\mathrm{Re}S)_H = (\mathrm{Re}S)_I$ and $(\mathrm{Re}T_i)_H = (\mathrm{Re}T_i)_I$.

On the heterotic side, the expression of l_H in terms of S and T_i is given by the linear-chiral relation (B.4), which in the particular case of the orbifold considered can be rewritten as $\frac{1}{l} = \mathrm{Re}S - \frac{b^H_{SU(12)}}{16\pi^2} \ln(V_H^{1/3} M_H^2)$, thus yielding:

$$\frac{1}{g_a^2(\mu^2)\,|_H} \;=\; \mathrm{Re}S \;-\; \frac{b^H_{SU(12)}}{16\pi^2} \ln\left(V_H^{1/3} M_H^2\right) \;-\; \frac{b^H_a}{16\pi^2} \ln \frac{\mu^2}{M_H^2} \;. \tag{34}$$

This is however not yet the relevant expression for the low-energy gauge couplings, since it holds in the trivial vacuum with a nonzero anomalous $U(1)$ D-term, $D_X = \xi_H^2$. Considering the physical flat direction with maximal gauge symmetry $G = SU(12) \times SO(8)$, along which ξ_H^2 is compensated for by vevs of the twisted moduli M_k, one finally finds:

$$\frac{1}{g_a^2(\mu^2)\,|_H} \;=\; \mathrm{Re}S \;-\; \frac{b^I_a}{16\pi^2} \ln\left(V_H^{1/3} \mu^2\right) \;. \tag{35}$$

The change from eq. (34) to eq. (35) is due to the fact that along the flat direction considered, some twisted charged states become massive and decouple, yielding $b^H_{SU(12)} \to b^I_a$ and $b^H_a \to b^I_a$. This results in a shift of the unification scale from the string scale to the compactification scale $M_c = V_H^{1/3}$.

On the orientifold side, using the dilaton linear-chiral duality relation (25), we obtain:

$$\frac{1}{g_a^2(\mu^2)\,|_I} \;=\; \mathrm{Re}S \;+\; s_a\, m \;-\; \frac{b^I_a}{16\pi^2} \ln \frac{\mu^2}{M_I^2} \;. \tag{36}$$

Unlike in the heterotic case, the Fayet-Iliopoulos term depends now on the twisted moduli [48, 54, 34] and the point of maximal gauge symmetry corresponds to $\xi_I^2 = 0$; as can be seen from an explicit linear-chiral duality transformation, ξ_I^2 is proportional to the twisted modulus m in the linear basis. Therefore, at the point of maximal symmetry, one has $m = 0$ and:

$$\frac{1}{g_a^2(\mu^2)\,|_I} \;=\; \mathrm{Re}S \;-\; \frac{b^I_a}{16\pi^2} \ln \frac{\mu^2}{M_I^2} \;, \tag{37}$$

which is obviously not dual to eq. (35). Note that, as was noticed in [36], duality would be restored if some (presumably nonperturbative) mechanism ensured that $\xi_I^2 = 0$ for $\mathrm{Re}M = 0$.

[8] Needless to say, there is a perfect matching of the gauge couplings of both models at tree-level. Indeed, at the point of maximal gauge symmetry, one has $m = 0$ (corresponding to $\xi_I^2 = 0$) on the orientifold side, so that $\frac{1}{g_a^2|_I} = \frac{1}{l_I}$ is indeed dual to $\frac{1}{g_a^2|_H} = \frac{1}{l_H}$.

[9] Recall that $(\mathrm{Re}S)_H = V_H M_H^6/\lambda_H^2$ and $(\mathrm{Re}T_i)_H = V_H^i M_H^2$ (where V^i denotes the volume of the compact 2-torus T^i) on the heterotic side, and $(\mathrm{Re}S)_I = V_I M_I^6/\lambda_I$ and $(\mathrm{Re}T_i)_I = V_I^i M_I^2/\lambda_I$ on the orientifold side [30].

4 More terms in the one-loop heterotic Lagrangian

4.1 One-loop corrections to the Kähler potential

We now consider specific one-loop corrections to the Kähler potential that are related to the well known holomorphic threshold corrections. In the remaining part of this paper we call these one-loop terms κ–corrections. Both types of corrections are uniquely correlated through the anomaly cancellation ten-dimensional Green-Schwarz terms [44].

Let us start with the perturbative heterotic string. As pointed out by Green and Schwarz for the gauge groups $E_8 \times E_8$ and $SO(32)$ the anomaly twelve form I_{12} factorizes as $I_4 X_8$ and all gauge, gravitational and mixed gauge-gravitational anomalies are cancelled by the variation of well defined local counterterms. The crucial role in this cancellation is played by the antisymmetric field B_{MN}, which must transform nontrivially under gauge transformations. To recover classical gauge invariance at tree level the strength of the antisymmetric tensor field is modified so that it is gauge invariant and obeys $dH = I_4$. In the Horava-Witten model [2] representing the low-energy effective theory of the strongly coupled heterotic $E_8 \times E_8$ string, where the two E_8 gauge sectors live on opposite boundaries of the eleven-diemnsional bulk, the role of the effective antisymmetric tensor fields participating in the Green-Schwarz mechanism is played by the components C_{AB11} of the three-form C. In general, there can be as many factorizable components of the anomaly form I_{D+2} in D dimensions, as many antisymmetric tensor fields are available in the model under considarations. A generalized Green-Schwarz mechanism involving more than one antisymmetric tensor field is at work in Type I/Type IIB orientifold models [24], [48]. The local Green-Schwarz counterterms must be added to the ten-dimensional action of the heterotic superstring, and must be taken into account when one performs the dimensional reduction/compactification down to four dimensions. Partial reduction of the GS terms has been performed in [55], [56], [57], and more recently in [3], [15] in the context of the strongly coupled heterotic string. In particular reference [57] contains essentially the complete result. Among the terms coming from this reduction the best known ones are the axionic parts of the holomorphic threshold corrections, $\pm\epsilon\,\theta F\tilde{F}$, where $\theta = Im(T)$, which have exactly the same form in the strongly coupled and in the weakly coupled heterotic $E_8 \times E_8$ string cases (they can be directly computed as the large T limit of the threshold corrections in weakly coupled orbifolds [4], [14]). However, those are not the only relevant terms that contribute one-loop terms to the four dimensional Lagrangian. The other ones, ignored so far, are terms involving derivatives of the matter fields. To see these terms arising from the compactification we need the explicit form of the GS counterterms in the case of the $E_8 \times E_8$ heterotic string. If we denote the anomaly associated with the twelve-form I_{12} by G, the new part in the 10d action satisfying $\delta_{gauge} S_{GS} + G = 0$ can be written as

$$S_{GS} = \int \left(4(\omega_{3L} - \frac{1}{30}\omega_{3Y})X_7 - 6BX_8\right) \tag{38}$$

where $dX_7 = X_8$ and $X_8 = \frac{1}{24}TrF^4 - \frac{1}{7200}(TrF^2)^2 - \frac{1}{240}TrF^2 trR^2 + \frac{1}{8}trR^4 + \frac{1}{32}(trR^2)^2$. To be specific, let us take the case of the standard embedding where $F^{(1)}_{MN} = R_{MN}$ and the Bianchi identity is fullfilled pointwise. The index on F indicates that we use the gauge connection of only one of the two E_8's, say the first one, to solve the Bianchi identity. As is well known, in that case the first E_8 is broken down to its E_6 subgroup, and the components of its connection with compact indices give rise to scalars A in $h_{1,1}$ **27** and $h_{1,2}$ $\bar{\mathbf{27}}$ representations of E_6. There are no matter fields associated with the unbroken E_8. The numbers $h_{a,b}$ are the Hodge numbers of the Calabi-Yau manifold which forms the compact 6d space. The usual axionic thresholds come from the terms which couple compact components of B, the B_{MN}, to the $F^{(i)}\tilde{F}^{(i)}$ term composed of 4d gauge field strengths. These come from the integral of $-6\,B\,X_8$ over the Calabi-Yau space. Noting that the expansion of the compact components of B in harmonics on the CY space is $B_{MN} = \sum_1^{h_{1,1}} \theta^Z(x)\Omega^Z_{MN}(y)$ where Ω^Z are the harmonic $(1,1)$ forms, the resulting 4d coupling is

$$L_\theta = \frac{1}{20}\theta^Z \left(F^{(1)}\tilde{F}^{(1)} \int_K \Omega^Z \wedge tr(F^{(1)} \wedge F^{(1)}) \;-\; F^{(2)}\tilde{F}^{(2)} \int_K \Omega^Z \wedge tr(F^{(2)} \wedge F^{(2)}) \right) \tag{39}$$

i.e. the couplings have exactly the same magnitude and opposite sign for E_6 and E_8 factors. The second coupling between 4d zero modes comes from the terms in S_{GS} which contain space-time components of B i.e. those proportional to $B_{\mu\nu}$. The physical degree of freedom associated with these components is the pseudoscalar dual to $H = dB + \omega_{3L} - \frac{1}{30}\omega_{3Y}$. After integration by parts one obtains the relevant part of the GS terms

$$S_{GS,\,H} = -6 \int_K H X^1_7 \tag{40}$$

where the standard embedding is assumed and $X^1_7 = \frac{1}{120}\omega^1_{3Y}(trF_1^2 - \frac{1}{2}trR^2)$. Let us note that if we were looking at corresponding terms with $\omega^{(2)}_{3Y}$ instead of $\omega^{(1)}_{3Y}$ then we would obtain the same expression with the opposite sign. The couplings of interest come from the terms

$$\epsilon^{\mu\nu\rho M\sigma NPQRS} H_{\mu\nu\rho} Tr(\mathcal{A}_M \partial_\sigma \mathcal{A}_N)(trF_1^2 - \frac{1}{2}trR^2)_{PQRS} \tag{41}$$

Assuming the Calabi-Yau space with $h_{1,2} = 0$, going over to complex coordinates $\mathcal{B}_1 = 1/\sqrt{2}(\mathcal{A}_4 + i\mathcal{A}_5), ..., \mathcal{B}_3 = 1/\sqrt{2}(\mathcal{A}_8 + i\mathcal{A}_9)$ and using the expansion [58] $\mathcal{B}_1 = T_{ax}A^{Kx}g^{a\bar{n}}\Omega^K_{1\bar{n}}, \bar{\mathcal{B}}_{\bar{1}} = \bar{T}_{\bar{a}\bar{x}}\bar{A}^{K\bar{x}}g^{\bar{a}n}\bar{\Omega}^K_{\bar{1}n}$, *etc.* with orthogonality relation for the gauge group generators $Tr(T_{ax}\bar{T}_{\bar{b}\bar{y}}) = g_{a\bar{b}}\delta_{x\bar{y}}$ one readily obtains the 4d coupling of the form

$$\delta L = \frac{i}{40}\epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho}(A^{Zx} \overleftrightarrow{\partial_\sigma} \bar{A}^{Y\bar{y}})\delta_{x\bar{y}} \int_K \sqrt{g_{(6)}}\epsilon^{MNPQRS} g^{TU}\Omega^Z_{MT}\Omega^Y_{NU}(trF_1^2 - \frac{1}{2}trR^2)_{PQRS} \tag{42}$$

In the next step one needs to use the duality relation between H and the universal axion D: $H_{\mu\nu\rho} = g_4^{-1/2}e^{-6\sigma}\phi^{3/2}\epsilon_{\mu\nu\rho\delta}\partial^\delta D$ where $g^{(10)}_{AB} = (e^{-3\sigma}g^{(4)}_{\mu\nu};\, e^\sigma g^{(0)}_{MN})$ and ϕ is related to the string dilaton[10]. Then one obtains the terms $\partial^\mu DA \overleftrightarrow{\partial_\mu} \bar{A}$ which give rise to kinetic mixing of the dilaton superfield S and untwisted matter superfields A. To find the relation between the integrals which are coefficients in the terms (42) and (39) we go now to the case $h_{1,1} = 1$. Then the cohomology group $H^{1,1}$ consists only of the Kähler class generated by the Kähler form $k = ig_{i\bar{j}}dz^i \wedge d\bar{z}^{\bar{j}}$ where $g_{i\bar{j}}$ is the CY metric. Going over to the holomorphic coordinates one can see that the integrand in (42) contains the factor

$$g^{m\bar{n}}k_{\bar{q}m}k_{p\bar{n}} = -g_{p\bar{q}} = ik_{p\bar{q}} \tag{43}$$

[10]We recall that definitions of the universal moduli complex scalars in the weakly coupled string convention are: $S = e^{3\sigma}\phi^{-3/4} + 3i\sqrt{2}D$ and $T = e^\sigma\phi^{3/4} + i\sqrt{2}\theta$.

which finally allows one to write the coefficient of the 4d operator in (42) in the form of the topological integral $-\frac{1}{2}\int_K k\wedge tr(F^{(1)}\wedge F^{(1)})$ which is the same as the integral in the axionic threshold formula (39). The simple derivation we have given here is equivalent to Witten's truncation on the six-torus when one replaces $g_{m\bar{n}}\to\delta_{m\bar{n}}$ and $k_{m\bar{n}}\to i\delta_{m\bar{n}}=\epsilon_{m\bar{n}}$. One should notice that the antisymmetric symbol ϵ introduced above reduces to the usual 2d antisymmetric symbol for each two-plane of the torus in the real coordinates.

The above field-theoretical model calculation establishes the intimate relation between the axionic threshold correction (which is imaginary part of the holomorphic threshold correction) and the nonholomorphic correction (42): either both of them vanish or both are present in the 4d effective Lagrangian. In terms of the four dimensional chiral superfields the nonholomorphic correction leads to the modified Kähler potential

$$K_S=-\log(S+\bar{S}-\kappa A\bar{A}) \tag{44}$$

This mixing with the 4d dilaton superfield is expected to hold for all untwisted matter multiplets, including the case of $h_{1,2}\neq 0$, in orbifold compactifications and is also valid in the case of nonstandard embeddings of the gauge group. The corrections to the Kähler potential arise obviously at one-loop order in the string coupling, since they come from the ten-dimensional Green-Schwarz terms. The sign of the mixing terms depends on the gauge group under consideration: it would be different for matter coming from the breaking of the second E_8. The same effect is present in the context of the Horava-Witten model which describes low-energy behaviour of the strongly coupled heterotic $E_8\times E_8$ superstring. Let us complete this argument by summarizing the relevant calculation in the Horava-Witten model. In that case the decomposition of the eleven-dimensional metric which leads to canonical Einstein-Hilbert terms in the four-dimensional action and exhibits relevant four-dimensional degrees of freedom is $g^{(11)}_{AB}=(e^{-2\beta(x)-\gamma(x)}g^{(4)};e^{-2\beta(x)+2\gamma(x)};e^{\beta(x)}g^{(0)}_{IJ})$. In this expression $e^{3\beta}$ is the fluctuating volume V of the 6d Calabi-Yau space in units of the reference volume $V_0=\int_K\sqrt{g^{(0)}}$. The real parts of the chiral moduli superfields in the effective action are $Re(S)=e^{3\beta}$ and $Re(T)=e^{\gamma}$. One should note, that the 10d source of kinetic terms for the 4d gauge fields and for 4d scalars is the operator $\frac{1}{8\pi(4\pi k_{11}^2)^{2/3}}\int d^{10}x\sqrt{g}TrF_{AB}F^{AB}$. The relevant parts of it are

$$\frac{1}{8\pi(4\pi k_{11}^2)^{2/3}}\int d^{10}x\sqrt{g}TrF_{\mu\nu}F^{\mu\nu}+\frac{1}{4\pi(4\pi k_{11}^2)^{2/3}}\int d^{10}x\sqrt{g}TrF_{\mu M}F^{\mu M} \tag{45}$$

where μ,ν are noncompact and M the compact indices, and k_{11}^2 is the eleven-dimensional gravitational constant. Using the decomposition of the gauge fields $\mathcal{A}_M$ given below formula (40) and the decomposition of the 11d metric in terms of β, γ one obtains

$$L_{kin}=\frac{V_0}{8\pi(4\pi k_{11}^2)^{2/3}}\int d^4x\sqrt{g^{(4)}}e^{3\beta}TrF_{\mu\nu}F^{\mu\nu}+\frac{V_0}{2\pi(4\pi k_{11}^2)^{2/3}}\int d^4x\sqrt{g^{(4)}}\frac{6}{e^{\gamma}}|\partial_\mu A|^2\,. \tag{46}$$

As pointed out in [2] computing corrections to the effective Lagrangian in the linear order in perturbations of the metric of the compact six-dimensional space corresponds to substitution $V=e^{3\beta}V_0\to V_{v,h}=(e^{3\beta}\pm\xi_0e^{\gamma})V_0$ at the visible and hidden walls respectively. The parameter ξ_0 is given by the topological integral

$$\xi_0\;=\;-\frac{\pi\rho_0}{2(4\pi)^{4/3}k_{11}^{2/3}}\frac{1}{8\pi^2}\int_X k\wedge(\,trF^{(1)}\wedge F^{(1)}-\frac{1}{2}\,trR\wedge R\,). \tag{47}$$

The normalization that gives direct correspondence with the weakly coupled case is established through $V_0 = 2\pi(4\pi k_{11}^2)^{2/3}$ and $\alpha' = \frac{1}{(4\pi)^{2/3}\pi^2}\frac{k_{11}^{2/3}}{\rho_0}$ where one typically puts $\alpha' = 1/2$. In the kinetic terms of the gauge fields this leads to the well known difference between gauge couplings (gauge kinetic functions) on different walls, and upon substitution into the second term in (45) this gives immediately the corrections to the kinetic terms for scalars[11]

$$L_{kin} = \frac{|\partial_\mu A|^2}{T+\bar{T}} \pm \xi_0 \frac{|\partial_\mu A_{v,h}|^2}{S+\bar{S}}. \tag{48}$$

These corrections have opposite sign for matter on opposite walls, and are interpreted as corrections to the S-dependent part of the 4d Kähler potential

$$K_S = -\log(S+\bar{S}) \pm \xi_0 \frac{A_{v,h}\bar{A}_{v,h}}{S+\bar{S}} \approx -\log(S+\bar{S} \mp \xi_0 A_{v,h}\bar{A}_{v,h}) \tag{49}$$

which is exactly the same expression that we have obtained in the weakly coupled heterotic string for matter living on the visible wall (one should observe that with the standard normalization of the integrals $\xi_0 = \kappa$). As we can see clearly in the context of the Horava-Witten model these corrections are indeed uniquely related to the corrections to the holomorphic gauge kinetic functions $f_{v,h} = S \pm \xi_0 T$. They both have here the geometric interpretation of the correction to the volume of the 6d compact space induced by different vacuum gauge fluxes on different walls. In the weakly coupled regime the same effects are seen as one-loop quantum effects, and from the point of view of the four-dimensional effective theory the matching is exact. The reason for this matching is anomaly cancellation. The crucial observation is that the massless spectrum of weakly and strongly coupled theories is the same, and, hence, that ten-dimensional and eleven-dimensional Green-Schwarz terms participating in anomaly cancellation must be strictly related to each other. The most relevant term in the compactification down to four dimensions is the topological term $C \wedge G \wedge G$ of eleven dimensional Horava-Witten Lagrangian, where C is the eleven-dimensional three-form field and G is its modified strength. It turns out that compactification[12] of the components with the index structure $\epsilon^{\mu\nu\rho\delta\, IJKL\, 11MN} G_{\mu\nu\rho\delta} G_{IJKL} C_{11MN}$ produces in four dimensions axionic parts of the threshold corrections

$$\begin{aligned}
\delta L^{(4)} = {} & \tfrac{1}{k_{11}^2}\tfrac{\rho_0}{24\pi}\left(\tfrac{k_{11}}{4\pi}\right)^{4/3} tr(F^{(1)}\tilde{F}^{(1)}) \left[\theta^Z \textstyle\int_X \Omega^Z \wedge (tr(F^{(1)} \wedge F^{(1)}) - \tfrac{1}{2}tr(F^{(2)} \wedge F^{(2)}) \right. \\
& \left. - \tfrac{1}{4}tr(R \wedge R))\right] \\
& + \tfrac{1}{k_{11}^2}\tfrac{\rho_0}{24\pi}\left(\tfrac{k_{11}}{4\pi}\right)^{4/3} tr(F^{(2)}\tilde{F}^{(2)}) \left[\theta^Z \textstyle\int_X \Omega^Z \wedge (tr(F^{(2)} \wedge F^{(2)}) - \tfrac{1}{2}tr(F^{(1)} \wedge F^{(1)}) \right. \\
& \left. - \tfrac{1}{4}tr(R \wedge R))\right].
\end{aligned} \tag{50}$$

These expressions, after substitution of ρ_0, coincide with the axionic threshold corrections given in (39). In the same way the results of the compactification of the part of the $C \wedge G \wedge G$ with the

[11] We remove the factor of 12 through rescaling of matter fields A.

[12] The compactification includes here substitution of the lowest order nontrivial solutions for G along the eleventh dimension.

index structure $\epsilon^{M\rho N\delta IJKL11\mu\nu} G_{M\rho N\delta} G_{IJKL} C_{11\mu\nu}$ coincide precisely with the $\partial^\mu DA \overset{\leftrightarrow}{\partial_\mu} \bar{A}$ terms given in (42) for matter in the visible sector (the sign would be the opposite one if we had matter in the hidden E_8 sector). Zero modes of the C_{11MN} and $C_{11\mu\nu}$ coincide with the axions which are zero modes of B_{MN} and $B_{\mu\nu}$. Thus we can see, that the relation between holomorphic threshold corrections and the κ-corrections to the Kähler potential receives even stronger support when viewed from the perspective of the strongly coupled heterotic models. There both types of 4d terms come from the topological term of the eleven dimensional supergravity, which is uniquely constrained by supersymmetry and by anomaly cancellation. Either both types of terms are present, or both should be absent in any heterotic model. With the present observation taken into account the 4d effective Lagrangians from the weakly and strongly coupled theories look exactly equivalent at the - respectively - one loop and linear in CY deformation orders[13].

Equally important from the point of view of the rest of the present paper is the observation, that it is natural to expect the corrections to K_S to have the same nature also in the heterotic $SO(32)$ models. The structure of the Green-Schwarz terms is in this case very much the same as described above and to be specific one can think of the decomposition $SO(32) \to SU(3) \times U(1) \times SO(26)$ where $SU(3)$ is identified with the holonomy group of the Calabi-Yau space in analogy with the calculation presented earlier in this section.

4.2 Modular transformations in the presence of κ-corrections to the Kähler function

As argued in the previous section in the (2,2) compactifications with holomorphic threshold corrections to the gauge couplings we expect the presence of nonholomorphic corrections which have a natural interpretation of one-loop contributions to the Kähler function. In what follows we restrict ourselves to the stndard embedding, with matter in the visible E_6 sector only. Then in the most symmetric case, with just the single T-modulus, the relevant parts of the Kähler function and the kinetic function are

$$K = -\log(S + \bar{S} + \frac{3\delta_{GS}}{4\pi^2}\log(T+\bar{T}) - \kappa A\bar{A}) - 3\log(T+\bar{T}) + \frac{A\bar{A}}{T+\bar{T}} \tag{51}$$

$$f = S - (\frac{6\kappa}{\pi} + \frac{3}{2\pi^2})\log\eta^2(T) + \sigma(T) \tag{52}$$

where κ is a computable numerical parameter and $\sigma(T)$ is the holomorhic part of universal one-loop threshold correction which is invariant under the $SL(2,Z)$ T-duality transformations and approaches $-\frac{1}{4\pi}T$ as $T \to \infty$. With the transformations $S \to S + \frac{3\delta_{GS}}{4\pi^2}\log(icT+d)$, $T \to \frac{aT-ib}{icT+d}$ the model given by the above K and f is no longer invariant at one loop. One can cancel the variation of the κ-term in K_S with a combination of the two modified transformations[14]:

[13] This concerns here models without five-branes in the 5d bulk on the strongly coupled side.

[14] This form of modified T-duality transformation is reminiscent of the situation in the (2,2) orbifold models when the moduli C, charged under the subgroup H from the decomposition $H \times E_6 \times E_8$ of the orbifold gauge group are to obtain an expectation value. Then the transformation of T is no longer an $SL(2,Z)$ transformation, but becomes extended in a holomorphic way by terms which can be represented in the form of the power-law expansion in the blowing-up moduli C: $T \to \frac{aT-ib}{icT+d} + g_n(T)C^n$, $n \geq 3$ [39]. One should notice, that when the duality transformation gets modified at tree level due to blowing up of the orbifold, either the transformation

- $S \to S + \frac{3\delta_{GS}}{4\pi^2}\log(icT+d) + \gamma_S\,\kappa A\bar{A}(\frac{1}{(icT+d)(-ic\bar{T}+d)} - 1)/2$,
- $T \to \tilde{\Gamma}T = \frac{aT-ib}{icT+d} - \gamma_T\,\frac{2\pi^2\kappa}{3\delta_{GS}}\frac{(icT+d)(-ic\bar{T}+d)-1}{(icT+d)^2(-ic\bar{T}+d)^2}(T+\bar{T})A\bar{A}$.

where $\gamma_S+\gamma_T=1$. Any combination of γ's is troublesome. First, let us note that the new terms should not be counted in $P_C K$ which enters the nonlocal term representing the triangle graphs with a Kähler or sigma-model connection, as that would be a higher order effect. Second, the new terms turn a chiral superfield into a general superfield. Let us look more closely at various possibilities. The new terms in the transformation of S do not spoil the anomaly cancellation, but they vary at one-loop order the physical, 1PI, gauge coupling constant, which is not consistent with the symmetry. These terms in the transformation of T are proportional to the ratio of two parameters κ and δ_{GS}. One can assume that this ratio is small and treat these new terms as one-loop contributions. Then one should drop it in the T-dependent threshold corrections to gauge couplings, but the Kähler function $K_T = -\log(T+\bar{T})$ becomes non-covariant, and there is no suitable term to restore covariance. One should observe for instance, that although the $T+\bar{T}$ and $|\eta(T)|^{-4}$ both transform identically with respect to the tree-level modular transformations, their derivatives are completly different, hence the higher order terms in the expansion in powers of $|A|^2$ of their images under the full transformations cannot be matched to restore the covariance.
To see this more explicitly lets us assume the Kähler potential for the superfield T in the form

$$K_T = -\log(T+\bar{T}+\beta|\eta(T)|^{-4}) \tag{53}$$

The requirement that the covariance is restored at one-loop gives a solution for the coefficient β (we put here $\gamma_T=1$ for convenience)

$$\beta = -\frac{1}{4\pi}\frac{|\eta(\Gamma T)|^4}{Re(G_2(\Gamma T))} \tag{54}$$

where $\Gamma T = (aT-ib)/(icT+d)$. This solution obviously does not make sense, as the coefficient β turns out to be a function depending on the parameters of the modular transformation. Let us note, that the form of the above solution suggests that one could try another counterterm,

$$K_T = -\log(T+\bar{T}+\frac{\beta}{|\hat{G}_2(T)|}) \tag{55}$$

where $\hat{G}_2$ is the covariant version of the Eisenstein form $G_2(T)$, transforming with modular weight 2. It is a straightforward calculation to find out that this form of the counterterm does not work, however, as under $\tilde{\Gamma}$

$$\hat{G}_2(T) \to \hat{G}_2(\tilde{\Gamma}T) = \hat{G}_2(\Gamma T)\left(1+\alpha A\bar{A}\left(\tfrac{5}{24}(icT+d)^2\tfrac{G_4(T)}{\hat{G}_2(T)}+\right.\right.$$
$$\left.\left.-\tfrac{1}{24}\left((icT+d)^2\tfrac{G_2^2(T)}{\hat{G}_2(T)} - \tfrac{4\pi^2c^2}{\hat{G}_2(T)} - 4\pi ic(icT+d)\tfrac{G_2(T)}{\hat{G}_2(T)} + \tfrac{4\pi}{(T+\bar{T})^2}\tfrac{(-ic\bar{T}+d)^2}{\hat{G}_2(T)}\right)\right)\right) \tag{56}$$

of S must get further modified at one loop, or the T-dependent counterterm under the logarithm must change to keep the $K(S,\bar{S})$ invariant.

with $\alpha = -\frac{2\pi^2\kappa}{3\delta_{GS}} \frac{(icT+d)(-ic\bar{T}+d)-1}{(icT+d)^2(-ic\bar{T}+d)^2}(T+\bar{T})$. Hence, once again, one obtains a coefficient β that depends explicitly on the transformation parameters.
Finally, one could ask whether, when we take $\gamma_S \neq 0$, in the gauge kinetic function f the new term due to transformation of S, δ_{AS}, would not cancel against the term δ_{AT} from the transformation of T. The point is that $\delta_{AS} \propto \delta_{GS}$ and $\delta_{AT} \propto \kappa^2/\delta_{GS}$, hence the terms are of different orders in loop expansion parameters. In other words, such cancellation would require certain conspiracy between κ and δ_{GS} – quantities which have very different microscopic origin and in principle do not need to be related. In addition, even a successful cancellation in f would not solve the problem of the covariance of K_T.

The problem which we have described in this section does not imply neccesarily that target-space duality does not hold in the heterotic string models. On the contrary, there are very good reasons to believe that it is an exact symmetry of many heterotic compactifications and as such should be representable in the effective Lagrangian. We would rather argue that there should exist further nonperturbative contributions to the Kähler function for moduli and matter fields. To illustrate these statements, let us recall that we have identified the κ–corrections to the kinetic terms in field theoretical limit, i.e. in the limiting domain of large $Re(T)$. In this limit the gauge kinetic function is $f = S \pm \kappa T$ (for E_6 and E_8 sectors) which is at odds with the usual form of T-duality. To establish T-duality one needs to promote T in the f to $\log \eta^2(T)$ plus $SL(2,Z)$-invariant universal terms. This is the necessary extension of f to the case of arbitrary, both large and small, values of $Re(T)$. Similar nontrivial extension of K_S is likely to be necessary to restore one-loop duality in the full effective Lagrangian, valid over the whole moduli space. The possible form of the generalized Kähler function could be

$$K(S,\bar{S};A,\bar{A}) = -\log(S+\bar{S}\ -\kappa A\bar{A}\,\frac{\log(j(T)-744)+\log(j(\bar{T})-744)}{2\pi(T+\bar{T})}) \tag{57}$$

where $j(T)$ is the $SL(2,Z)$-invariant form. With the expression (57) substituted into the Kähler function the Lagrangian becomes modular invariant at the one-loop level without the need to modify the standard T-duality transformation for moduli and matter fields. Also, in the large T limit it reduces to the expressions which we have computed in the section 4.1 . The expression (57) is not the only one which fulfills these conditions. There exist other possible choices[15], with a different behaviour at small T.

However, it still makes sense to discuss the effective Lagrangians with the perturbative form of K_S (including κ-terms) and to compare them to perturbative Lagrangians coming from other compactifications, having in mind that one restricts oneself to a part of moduli space where $Re(T) >> 1$ in natural units and compare terms that might violate T-duality.

The kinetic mixing between the dilaton modulus S and matter fields transforming like tensors under $SL(2,Z)$ leaves some questions concerning heterotic–type IIB orientifold duality.

[15]For instance:

$$K(S,\bar{S};A,\bar{A}) = -\log(S+\bar{S}\ -\kappa A\bar{A}\,|\eta(T)|^4\,|j(T)-744|^{1/6})\,,$$
$$K(S,\bar{S};A,\bar{A}) = -\log(S+\bar{S}\ -\kappa A\bar{A}\,|\hat{G}_2(T)|(\tfrac{1}{3}-\tfrac{4}{\log(j(T)-744)+\log(j(\bar{T})-744)})^{-1}/\pi^2)\,.$$

to name just two examples. Also, these are lowest order terms in the expansion in $A\bar{A}$. The exact form has to be determined by string calculations.

On one hand it spoils the one-loop duality invariance of the naive effective Lagrangian, thus making the situation more symmetric between the two classes of models - T-duality could be violated at one-loop level in a region of moduli space in both of the models. However, the way it would actually be violated seems to be completely different on both sides. Moreover, we recall, that we believe that in orbifold models κ–corrections are due to the existence of $N = 2$ subsectors. Hence, in this respect, making relations with orbifold models is justified in the analysis of the orientifolds like the Z_6' one (whose dual partner is not known). In addition, we do not expect similar matter-dilaton kinetic mixing in type IIB orientifold models. The reason is that there exists no one-loop coupling between the untwisted antisymmetric tensor field, corresponding to the imaginary part of the dilaton, and gauge fields. If the intuition gained during the analysis of the anomaly cancelling counterterms in heterotic models is correct, the kinetic mixing terms would rather be partners of the $B^{(k)} \wedge F^{(k)}$ couplings. Speculating further, by the same token we would expect modifications of the form $\frac{1}{4}(M_k + \bar{M}_k + \kappa_k A_k \bar{A}_k)^2$ to the Kähler functions of twisted moduli dual to twisted antisymmetric tensor fields, which have not been seen in explicit calculations.

5 Conclusions

Many of the properties of string theory can be understood through a study of symmetries. In view of possible phenomenological applications it is important to incorporate such symmetries (if possible) into the low-energy effective field theory actions. We have seen that target-space duality is a very useful symmetry, able to constrain severely those effective actions. In the framework of simple compactification schemes of the heterotic theories, target-space duality can be incorporated successfully. This even goes beyond the classical level and constrains the one-loop effective action, including a mechanism for cancellation of all target-space anomalies.

In the first part of the paper we recall the discussion of the simplest cases, Z_3 and Z_7 heterotic orbifolds, in detail. We then try to extend this picture to certain type IIB orientifold models (again Z_3 and Z_7), that have received some attention recently. One of the results of this paper is the observation that target-space duality now suffers from anomalies that cannot be cancelled by a mechanism similar to that in the heterotic case. The failure at the level of sigma-gravitational anomalies could in principle be repaired by T-dependent corrections to the CP-odd R^2 terms, but those would have to arise nonperturbatively. Our results (independently of the question of interpretation) strengthen the earlier arguments against a conjectured duality of a certain class of heterotic-type IIB orientifold models. Target-space dualities can be cancelled on the heterotic side, while no such satisfactory field theoretical cancellation mechanism seems to be at work in the considered Z_3 and Z_7 type IIB orientifolds.

Another face of the problem with T-duality, and consequently with heterotic–type IIB orientifold duality, shows up when one tries to interpret the threshold corrections that have been recently computed in Z_N orientifold models in the light of the cancellation mechanism suggested by the field-theoretical analysis of anomalies. In heterotic orbifolds, the structure of threshold corrections is intimately linked to the mechanism of mixed sigma-model-gauge anomaly cancellation, and one can actually infer from the explicit form of the one-loop corrections to gauge couplings how these anomalies are precisely compensated for. The interpretation of threshold corrections in orientifolds is less obvious due to the fact that the upper scale of logarithmic running is not modular invariant. Upon performing the linear-chiral duality transformation that relates the string result to the one-loop gauge couplings computed in the effective supergravity theory, one finds that this results in the impossibility to ensure target-space duality at the one-loop level: either sigma-gauge anomalies are cancelled, but the twisted moduli Kähler potential is not invariant; or the shift of the twisted moduli leaves an uncancelled anomaly. Of course, these results may simply point out that some nonperturbative terms in the orientifold Lagrangian are still missing, and in any case further string calculations in these models are truly needed. However, as far as one can trust the field theory approach, we have seen that target-space symmetries are a powerful tool to study and test conjectured relations between various string theories and the structure of their effective Lagrangians.

It is well established that on the heterotic side, a symmetry that corresponds to T-duality does exist at the level of string theory. Therefore, a suitable description of this symmetry should be possible at the level of the field theoretic low-energy effective Lagrangian. As we have seen earlier, this works without problems in the simplest models. In section 4 we report on an attempt to generalize this to more general cases (e.g. to those models which contain $N = 2$ subsectors). We point out that a manifestly symmetric incorporation of T-duality becomes problematic due to the appearance of one-loop corrections to the Kähler potential. Several possibilities to make these terms consistent with T-duality are proposed. Here the question arises, whether string theory can be approximated by a single unique low-energy effective action. After all, the traditional approach treated this action as relevant for the large T limit. Maybe different low-energy effective actions might be necessary to describe other patches of T moduli space. Nevertheless, we demonstrate that such a unique description might be possible for the heterotic models under consideration.

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Appendix A: Sigma-model anomalies in heterotic orbifolds

In this appendix, we recall some basic facts about sigma-model anomalies in $D = 4$, $N = 1$ heterotic orbifolds. At tree level, the Lagrangian is invariant under $SL(2, R)$ reparametrizations of the geometric moduli[16] [37, 38, 39],

$$T_i \;\rightarrow\; \frac{a_i T_i - i b_i}{i c_i T_i + d_i} \;, \qquad\qquad a_i d_i - b_i c_i = 1 \;, \tag{A.1}$$

together with linear transformations of the matter fields:

$$\Phi_\alpha \;\rightarrow\; \prod_{i=1}^{3} (i c_i T_i + d_i)^{n^i_\alpha}\, \Phi_\alpha \;, \tag{A.2}$$

where the n^i_α are the modular weights of the chiral superfield Φ_α. At the one-loop level, this symmetry known as target-space duality is generally anomalous; indeed, it acts as a chiral rotation on fermions and can have mixed anomalies with gauge symmetries and gravity[17]. Under an $SL(2, R)_{T_i}$ transformation, the one-loop Lagrangian undergoes an anomalous variation

$$\delta\mathcal{L}_{\text{anomaly}} \;=\; \frac{\theta_i}{32\pi^2} \sum_a b'^i_a\, F^a \widetilde{F}^a \;-\; \frac{\theta_i}{768\pi^2}\, b'^i_{grav}\, R\widetilde{R} \;, \tag{A.3}$$

where $\theta_i = \arg(i c_i T_i + d_i)$ is the angle of the chiral rotation, and the mixed sigma-gauge and sigma-gravitational anomaly coefficients b'^i_a and b'^i_{grav} are given by [41, 40]:

$$b'^i_a \;=\; -\, C_2(G_a) \,+\, \sum_\alpha (1 + 2n^i_\alpha)\, T(R_\alpha) \;, \tag{A.4}$$

$$b'^i_{grav} \;=\; 21 \;+\; 1 \;+\; b'^i_{mod} \;-\; \dim G \;+\; \sum_\alpha (1 + 2n^i_\alpha) \;. \tag{A.5}$$

In Eq. (A.4), $C_2(G_a)$ is the quadratic Casimir of the gauge group G_a and $T(R_\alpha)$ is the index of the representation R_α of G_a; in Eq. (A.5), $\dim G$ is the dimension of the total gauge group, 21 and 1 stand for the contribution of the gravitino and the dilatino respectively, and b'^i_{mod} denotes the contribution of the other (gauge singlet) modulinos, which is model-dependent. The mixed sigma-gauge anomaly is reproduced by the variation of the following nonlocal term [42, 41]:

$$\mathcal{L}_{\text{n.l.}} \;=\; -\,\frac{1}{32\pi^2} \sum_a \int \mathrm{d}^2\theta\; W^a W^a \sum_i b'^i_a\; P_C \left[\ln(T_i + \bar{T}_i) \right] \;+\; \text{h.c.} \;, \tag{A.6}$$

where $P_C = -\frac{1}{16}\, \Box^{-1} \bar{D}^2 D^2$ is the chiral projector, defined such that $\bar{D}\,(P_C H) = 0$ for any superfield H, and $P_C H = H$ if H is a chiral superfield. In terms of components, $\mathcal{L}_{\text{n.l.}}$ also contains a non-harmonic contribution to gauge couplings:

$$-\,\frac{1}{4} \sum_a F^a F^a \left(-\sum_i \frac{b'^i_a}{16\pi^2}\; \ln(T_i + \bar{T}_i) \right) \;. \tag{A.7}$$

[16] We consider only the diagonal moduli T_i, $i = 1, 2, 3$, which are common to all orbifolds. In the presence of off-diagonal moduli $T_{i\bar{j}}$ $(i \neq j)$, the target-space duality group is actually larger than $\prod_{i=1}^{3} SL(2, R)_{T_i}$.

[17] Strictly speaking, as explained in subsection 2.2, target-space duality transformations combine a sigma-model reparametrization and a Kähler transformation, both of which are anomalous [41]. Following the literature, we shall call the resulting anomalies (A.4) and (A.5) indifferently "target-space duality anomalies" or "sigma-model anomalies".

This shows that non-harmonic one-loop corrections to the gauge couplings are related, through supersymmetry, to the presence of sigma-model anomalies.

Since the discrete version of the above symmetries is related to T-duality, which is an exact symmetry of the heterotic string, these anomalies must be compensated in some way. Two mechanisms can be at work [41]: (i) a Green-Schwarz mechanism [44] similar to the one responsible for the cancellation of abelian gauge anomalies [45], which involves a non-linear transformation of the dilaton superfield at the one-loop level:

$$S \;\to\; S \;-\; \frac{1}{8\pi^2}\,\delta^i_{GS}\ln(ic_iT_i+d_i)\ ; \tag{A.8}$$

(ii) non-invariant, T_i-dependent holomorphic corrections to the gauge kinetic function. Such corrections arise from loops of massive string states and are associated with complex planes that are left invariant by some of the orbifold twists (corresponding to $N=2$ subsectors of the orbifold). In a large class of models, they take the form [59]

$$\Delta f_a^{1-\mathrm{loop}} \;=\; -\frac{1}{4\pi^2}\sum_i c_{a,i}\,\ln\left[\eta(T_i)\right]\ , \tag{A.9}$$

where the coefficient $c_{a,i}$, to be determined by an explicit string computation, vanishes when the i^{th} complex plane is rotated by all twists. Note that T_i-dependent threshold corrections explicitly break the continuous $SL(2,R)_{T_i}$ symmetry to its discrete version $SL(2,Z)_{T_i}$, while the Green-Schwarz mechanism preserves it. Under an $SL(2,Z)_{T_i}$ transformation, one has $\eta^2(T_i)\to(ic_iT_i+d_i)\,\eta^2(T_i)$ and

$$f_a^{1-\mathrm{loop}} \;\to\; f_a^{1-\mathrm{loop}} \;-\; \frac{1}{8\pi^2}\left(\delta^i_{GS}+c_{a,i}\right)\ln(ic_iT_i+d_i)\ . \tag{A.10}$$

Anomaly cancellation occurs provided the following relations are satisfied:

$$b'^i_a \;=\; \delta^i_{GS} \;+\; c_{a,i}\ . \tag{A.11}$$

Since $c_{a,i}=0$ when the i^{th} complex plane is rotated by all twists, Eq. (A.11) imposes a strong constraint on the corresponding sigma-model anomalies, which must then be gauge-group independent (exactly as happens for abelian gauge anomalies compensated by a Green-Schwarz mechanism). Cancellation of mixed sigma-gravitational anomalies is realized in a similar manner.

Collecting all contributions to the one-loop effective Lagrangian for the gauge fields, Eq. (A.6) and (A.9), and using Eq. (A.11), one obtains [41]:

$$\begin{aligned}\mathcal{L}_{\mathrm{gauge}} \;=\; \frac{1}{4}\sum_a\int\mathrm{d}^2\theta\; W^aW^a\;P_C\Big\{&\Big[S+\bar S\;-\;\frac{1}{8\pi^2}\,\delta^i_{GS}\ln(T_i+\bar T_i)\Big]\\ &-\frac{1}{8\pi^2}\sum_i(b'^i_a-\delta^i_{GS})\,\ln\Big[|\eta(T_i)|^4(T_i+\bar T_i)\Big]\Big\}\;+\;\mathrm{h.c.}\ .\end{aligned} \tag{A.12}$$

Eq. (A.12), together with Eq. (B.5), shows that the structure of one-loop corrections to the effective string action is strongly constrained by target-space duality anomaly cancellation.

Note that anomaly considerations do not tell us anything about possible holomorphic, modular invariant corrections [4] that are not included in Eq. (A.12).

Appendix B: Green-Schwarz mechanism in the linear multiplet formalism

The Green-Schwarz mechanism that cancels part of the sigma-model anomalies in heterotic orbifolds can be naturally described in the linear multiplet formalism[18]. Indeed, in terms of the string massless states, the axion-dilaton-dilatino system fits into a linear multiplet $L = (l, B_{\mu\nu}, \chi)$, where the antisymmetric two-tensor $B_{\mu\nu}$ is dual to the model-independent axion $a = \mathrm{Im}S$, $\partial_\mu a \sim \epsilon_{\mu\nu\rho\sigma}\,\partial^\nu B^{\rho\sigma}$. L couples to the gauge fields in such a way that the combination $\widehat{L} = L - 2\Omega$, where Ω is the Chern-Simons superfield defined by $\bar{D}^2\Omega = \sum_a W^a W^a$ and $D^2\Omega = \sum_a \bar{W}^a \bar{W}^a$, is gauge invariant. A gauge-invariant Lagrangian for L then takes the simple form $\mathcal{L}_L = \int \mathrm{d}^4\theta\ \Phi(\widehat{L})$. The transformation to the dual formulation in terms of the dilaton chiral superfield is accomplished by treating $\widehat{L}$ as an unconstrained superfield and adding the constraint $\bar{D}^2(\widehat{L} + 2\Omega) = D^2(\widehat{L} + 2\Omega) = 0$ to the Lagrangian:

$$\mathcal{L} \ = \ \int \mathrm{d}^4\theta \ \left[\Phi(\widehat{L}) \ - \ \frac{1}{2}\,(S + \bar{S})\,(\widehat{L} + 2\Omega) \right] \ , \tag{B.1}$$

where S is the dilaton chiral superfield, and an unconstrained superfield Σ defined by $S = \bar{D}^2\Sigma$ plays the role of the Lagrange multiplier. The equation of motion for Σ is nothing else but the constraint for $\widehat{L}$, while the equation of motion for $\widetilde{L}$ gives the duality relation:

$$\frac{\partial \Phi}{\partial \widehat{L}} \ = \ \frac{1}{2}\,(S + \bar{S}) \qquad \Rightarrow \qquad \widehat{L} \ = \ \widehat{L}\,(S, \bar{S}) \ . \tag{B.2}$$

Putting (B.2) into (B.1), one obtains the Lagrangian for S, $\mathcal{L}_S = \int \mathrm{d}^4\theta\, K(S, \bar{S}) + \frac{1}{4}\int \mathrm{d}^2\theta\, f(S)$, with $f(S) = S$ and the Kähler potential given by:

$$K\,(S, \bar{S}) \ = \ \left[\Phi(\widehat{L}) \ - \ \frac{1}{2}\,(S + \bar{S})\,\widehat{L} \right]_{\widehat{L} \,=\, \widehat{L}\,(S, \bar{S})} \ . \tag{B.3}$$

At tree level, one has $\Phi(\widehat{L}) = \ln \widehat{L}$, which leads to the duality relation $1/\widehat{L} = (S + \bar{S})/2$ and the Kähler potential $K(S, \bar{S}) = -\ln(S + \bar{S})$. The Green-Schwarz terms needed for the cancellation of sigma-model anomalies come at one loop and take the form $\Delta\mathcal{L}_{GS} = \sum_i \frac{\delta^i_{GS}}{16\pi^2}\,\widehat{L}\,\ln(T_i + \bar{T}_i)$. In components, this contains a coupling between the antisymmetric tensor $B_{\mu\nu}$ and the sigma-model connection. Through a linear-chiral duality transformation, $\Delta\mathcal{L}_{GS}$ translates into a one-loop mixing between the dilaton and the geometric moduli. Indeed, the one-loop duality relation reads

$$\frac{1}{\widehat{L}} \ = \ \frac{1}{2} \left[S + \bar{S} \ - \ \frac{1}{8\pi^2} \sum_i \delta^i_{GS} \ln(T_i + \bar{T}_i) \right] \ , \tag{B.4}$$

[18] For a useful reference about the linear multiplet in effective heterotic string theories, see [50] and references therein.

implying a Kähler potential of the form (the dots refer to further corrections that are not related to sigma-model anomaly cancellation):

$$K^{1-\text{loop}}\,(S,\bar{S}) \;=\; -\ln\left[S+\bar{S} \;-\; \frac{1}{8\pi^2}\,\sum_i \delta^i_{GS}\ln(T_i+\bar{T}_i) \;+\; \dots\right]\,. \tag{B.5}$$

Target-space modular invariance then requires that S transforms at one loop according to (A.8).

In type IIB orientifold compactifications we need to add more linear multiplets m_k, $k = 1,...,n_f$ where f is the number of twisted sectors. Such an extension of the linear multiplet formalism becomes somewhat subtle in the context of supergravity. To illustrate the trouble, let us take the simple case of just a single additional linear multiplet. If one neglects the superpotential couplings, the Lagrangian is given by $\mathcal{L} = S_0\bar{S}_0\Phi(\hat{L},\hat{m})$ where $S_0 = (z_0,\psi_0,f_0)$ is the conformal compensator. Let us take

$$\Phi = \frac{1}{\sqrt{2}}e^{K/3}(-X^{-1/2} + \frac{s}{2}X^{1/2}Y^2)\ , \tag{B.6}$$

with $X = e^{K/3}\frac{\hat{L}}{S_0\bar{S}_0}$, $Y = e^{K/3}\frac{\hat{m}}{S_0\bar{S}_0}$. Using this form of Φ one obtains the graviton kinetic term in the form

$$-\frac{1}{2}R\,(\frac{|z_0|^3}{(2e^K l)^{1/2}} + \frac{sm^2}{4}\frac{(2e^K l)^{1/2}}{|z_0|^3})\ . \tag{B.7}$$

It is clear that choosing the value of z_0 which corresponds to the Einstein frame, $|z_0|^3 = (1-\frac{s}{4}m^2)(2e^K l)^{1/2}$ for small m, leads in general to a mixing between m and l. The effective Lagrangian becomes simple and similar in its form to the heterotic effective Lagrangian only in the vicinity of the point $<m>= 0$. There the choice (B.6) gives in the leading order in m the gauge coupling $\frac{1}{g^2} = \frac{1}{l} + s\,m$ and the quadratic form of the Kähler potential for the dual chiral field M: $K \sim (M+\bar{M})^2 + \dots$. In this regime it is also possible to take into account one-loop corrections to $1/g^2$, e.g. through $\delta_{(1)}\Phi \sim \frac{3b_0}{4}\frac{\hat{m}}{S_0\bar{S}_0}\log(X)$ which produces $\sim b_0\log(l)$ in $1/g^2$.

References

[1] E. Witten, Nucl. Phys. B471 (1996) 135.

[2] P. Horava and E. Witten, Nucl. Phys. B460 (1996) 506; P. Horava and E. Witten, Nucl. Phys. B475 (1996) 94.

[3] T. Banks and M. Dine, Nucl. Phys. B479 (1996) 173.

[4] H.P. Nilles and S. Stieberger, Nucl. Phys. B499 (1997) 3.

[5] I. Antoniadis, M. Quiros, Nucl. Phys. B505 (1997) 109.

[6] E. Dudas, Ch. Grojean, Nucl. Phys. B507 (1997) 553.

[7] H.P. Nilles, M. Olechowski and M. Yamaguchi, Phys. Lett. B415 (1997) 24.

[8] Z. Lalak and S. Thomas, Nucl. Phys. B515 (1998) 55.

[9] A. Lukas, B. Ovrut, D. Waldram, Nucl. Phys. B532 (1998) 43.

[10] E. A. Mirabelli and M. E. Peskin, Phys. Rev. D58 (1998) 65002.

[11] H.P. Nilles, M. Olechowski and M. Yamaguchi, Nucl. Phys. B530 (1998) 43.

[12] A. Lukas, B. A. Ovrut, K.S. Stelle, D. Waldram, Phys. Rev. D59 (1999) 86001.

[13] J. Ellis, Z. Lalak, S. Pokorski and W. Pokorski, Nucl. Phys. B540 (1999) 149.

[14] S. Stieberger, Nucl. Phys. B541 (1998) 109.

[15] Z. Lalak, S. Pokorski and S. Thomas, Nucl. Phys. B549 (1999) 63.

[16] J. Ellis, Z. Lalak and W. Pokorski, eprint hep-ph/9811133, Nucl. Phys. B in press.

[17] K. Meissner, H.-P. Nilles and M. Olechowski, eprint hep-th/9905139, Nucl. Phys. B in press.

[18] J. Ellis, Z. Lalak, S. Pokorski, S. Thomas, eprint hep-th/9906148, Nucl. Phys. B in press.

[19] T. Li, Phys. Rev. D59 (1999) 107902.

[20] Z. Lalak, S. Thomas, eprint hep-th/9908147.

[21] A. Lukas, K.S. Stelle, preprint OUTP-99-49P, eprint hep-th/9911156.

[22] A. Sagnotti, in Cargese '87, "Non-Perturbative Quantum Field Theory", eds. G. Mack et al. (Pergamon Press, Oxford, 1988), p. 251; P. Horava, Nucl. Phys. B327 (1989) 461, Phys. Lett. B231 (1989) 251; J. Dai, R. Leigh and J. Polchinski, Mod. Phys. Lett. A4 (1989) 2073; R. Leigh, Mod. Phys. Lett. A4 (1989) 2767.

[23] E. G. Gimon, Joseph Polchinski, Phys. Rev. D54 (1996) 1667.

[24] C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti, and Ya.S. Stanev, Phys. Lett. B385 (1996) 96.

[25] Z. Kakushadze, Nucl. Phys. B512 (1998) 221.

[26] Z. Kakushadze and G. Shiu, Phys. Rev. D56 (1997) 3686.

[27] Z. Kakushadze, G. Shiu and S.-H.H. Tye, Nucl. Phys. B533 (1998) 25.

[28] Z. Kakushadze and G. Shiu, Nucl. Phys. B520 (1998) 75.

[29] L.E. Ibanez, JHEP 9807 (1998) 002.

[30] G. Aldazabal, A. Font, L.E. Ibanez, and G. Violero, Nucl. Phys. B536 (1998) 29.

[31] J. Lykken, E. Poppitz, S. P. Trivedi, Nucl. Phys. B543 (1999) 105.

[32] L.E. Ibanez, C. Munoz and S. Rigolin, preprint FTUAM-98-28, eprint hep-ph/9812397.

[33] M. Cvetic, L. Everett, P. Langacker and J. Wang, preprint UPR-0831T, eprint hep-th/9903051.

[34] Z. Lalak, S. Lavignac and H.P. Nilles, Nucl. Phys. B559 (1999) 48.

[35] J. Polchinski and E. Witten, Nucl. Phys. B460 (1996) 525.

[36] I. Antoniadis, C. Bachas and E. Dudas, Nucl. Phys. B560 (1999) 93.

[37] S. Ferrara, D. Lüst, A. Shapere, S. Theisen, Phys. Lett. B225 (1989) 363.

[38] J. Lauer, J. Mas and H. P. Nilles, Phys. Lett. B226 (1989) 251; Nucl. Phys. B351 (1991) 353.

[39] S. Ferrara, D. Lüst, S. Theisen, Phys. Lett. B242 (1990) 39.

[40] L. E. Ibanez, D. Lüst, Nucl. Phys. B382 (1992) 305.

[41] J.-P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B372 (1992) 145.

[42] G. L. Cardoso, B. A. Ovrut, Nucl. Phys. B369 (1992) 351.

[43] G. L. Cardoso, B. A. Ovrut, Nucl. Phys. B392 (1993) 315.

[44] M. Green and J. Schwarz, Phys. Lett. B149 (1984) 117.

[45] M. Dine, N. Seiberg, E. Witten, Nucl. Phys. B289 (1987) 589.

[46] I. Antoniadis, E. Gava, K.S. Narain, T.R. Taylor, Nucl. Phys. B407 (1993) 706.

[47] L.E. Ibanez, R. Rabadan and A.M. Uranga, preprint FTUAM-99-14, eprint hep-th/9905098.

[48] L.E. Ibanez, R. Rabadan and A.M. Uranga, Nucl. Phys. B542 (1999) 112.

[49] A. Sagnotti, Phys. Lett. B294 (1992) 196.

[50] J.-P. Derendinger, F. Quevedo and M. Quiros, Nucl. Phys. B428 (1994) 282.

[51] M. Klein, preprint FTUAM-99-34, eprint hep-th/9910143.

[52] C.A. Scrucca, M. Serone, eprint hep-th/9912108.

[53] V. Kaplunovsky and J. Louis, Nucl. Phys. B422 (1994) 57.

[54] E. Poppitz, Nucl. Phys. B542 (1999) 31.

[55] J.P. Derendinger, L.E. Ibanez, H.P. Nilles, Nucl. Phys. B267 (1986) 365.

[56] L.E. Ibanez, H.P. Nilles, Phys. Lett. B169 (1986) 354.

[57] H. Itoyama and J. Leon, Phys. Rev. Lett. 56 (1986) 2352.

[58] E. Witten, Phys. Lett. B155 (1985) 151.

[59] L.J. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. B355 (1991) 649.